

Bottomonium in a Bethe-Salpeter-equation study

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Using a well-established effective interaction in a rainbow-ladder truncation model of QCD, we fix the remaining model parameter to the bottomonium ground-state spectrum in a covariant Bethe-Salpeter equation approach and find surprisingly good agreement with the available experimental data including the $2^{--} \Upsilon(1D)$ state. Furthermore, we investigate the consequences of such a fit for charmonium and light-quark ground states.

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I. INTRODUCTION

In QCD, mesons appear as bound states of (anti)quarks and gluons. The bottomonium system, in particular below the $B\bar{B}$ threshold, is a prototype for the successful description of a meson using a straightforward $\bar{q}q$ -picture (for reviews on the subject, see e.g. the comprehensive compilations of the quarkonium working group [1, 2]). Such a simple setup is expected to be generally more realistic and accurate for heavier than for light quarks. In a much similar way, in a covariant Bethe-Salpeter-equation (BSE) approach a simple truncation like the well-established rainbow-ladder (RL) truncation is expected to be more accurate for heavier quarks and their bound states. We perform a basic initial test for this hypothesis by employing a model historically set up to describe light mesons. In this way, we study ground-state mesons for spins $J = 0, 1, 2$ starting with bottomonium down to light quarks and check the possibility to arrive at a reasonable agreement with experiment without fine-tuning. The latter point is important, since a general result or trend must be visible before one optimizes or fine-tunes the available model parameters.

The paper is organized as follows: in Secs. II and III we review the necessary details of the approach and the interaction. Section IV deals with the bottomonium ground states, followed by an investigation of the consequences of our parameter choice for lower quark masses in Sec. V. We conclude and present an outlook in Sec. VI. All calculations have been performed using Landau-gauge QCD in Euclidean momentum space.

II. MESONS IN AN RL MODEL OF QCD

We employ QCD's Dyson-Schwinger-equations (DSEs) (see, e.g. [3, 4] for recent reviews) coupled with the quark-antiquark Bethe-Salpeter equation (BSE). The latter is the covariant bound-state equation for the study of mesons in this context [5–7], and analogously one can

use a covariant approach to baryons in both a quark-diquark picture (e.g. [8–10] and references therein) or a three-quark setup [11, 12].

Numerical hadron studies such as the present one make a truncation of this infinite tower of coupled and in general nonlinear integral equations necessary. Herein, we use the so-called rainbow-ladder (RL) truncation, which is well-established as a tool for modeling hadron physics in QCD. In particular, it is better-suited as an approximation to the full set of equations the higher the quark mass becomes, see e.g., [13–15]. Related concrete results on the heavy-quark domain and -limit of Coulomb-gauge QCD have become available recently [16–18].

The RL truncation is simple yet offers the possibility for sophisticated model studies of QCD within the DSE-BSE context, since it satisfies the relevant (axial-vector and vector) Ward-Takahashi identities (see e.g. [19–26]). Regarding the meson spectrum a generally accurate description on the basis of a purely phenomenologically oriented model is conceivable. However, to increase the predictive power of the model it is advisable to reduce the free parameters in such a model, or more precisely in the effective model interaction, as much as possible.

The expectation in such a situation is that the description of light mesons will not be as accurate, which is a consequence of the fact that additional terms in the dressed quark-gluon vertex, which are omitted in RL truncation, cannot be successfully mimicked by a simple parametrization of the effective interaction such as the one used here. In the present study however, this is a defect we are willing to accept in order to check the validity of our assumptions about the heavy-quark domain. Our restrictions are further justified by meson studies beyond RL truncation (see, e.g., [27, 28] for relevant references) that have confirmed effects from correction terms, but at the same time shown both the numerical complexity of such investigations as well as the uncertainty of the size of even further corrections.

In RL truncation the axial-vector Ward-Takahashi identity dictates that the rainbow-truncated integral-equation kernel of the quark DSE corresponds to the ladder-truncated integral-equation kernel of the quark-antiquark BSE as given below. The identity is crucial to correctly realize chiral symmetry and its dynamical breaking in the model calculation from the very be-

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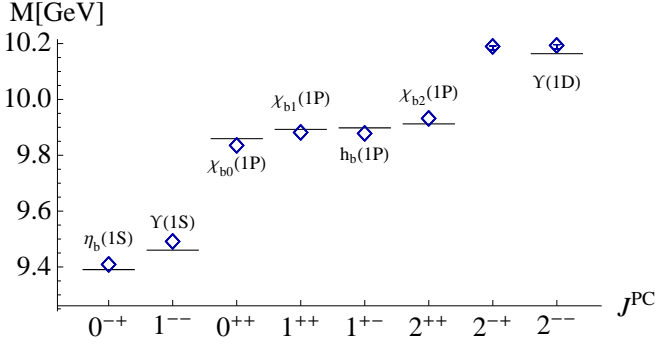


FIG. 1. (Color online) The bottomonium ground-state spectrum compared to experimental data.

ginning. As the most prominent result, one satisfies Goldstone’s theorem [23] and obtains a generalized Gell-Mann–Oakes–Renner relation valid for all pseudoscalar mesons and all current-quark masses [29, 30]. This relation can also be checked numerically and is satisfied at the per-mill level in our calculations.

We start out from a model setup defined in Ref. [31] and given in detail below, which has since been successfully applied to many in particular pseudoscalar- and vector-meson properties in recent years (see e.g. [4, 32] for comprehensive bibliographies). Of interest are electromagnetic hadron properties [25, 33–36], strong hadron decay widths [37], valence-quark distributions of pseudoscalar mesons [38–40], a study of tensor mesons [41] and an exploratory application of this model to the chiral phase transition of QCD at finite temperature [42]. Of immediate interest are the recent steps to the successful numerical treatment of heavy quarks in this particular model setup [32, 43–45].

III. QUARK DSE AND MESON BSE

In RL truncation one considers a meson with total $q\bar{q}$ momentum P and relative $q\bar{q}$ momentum q by consistently solving the homogeneous, ladder-truncated $q\bar{q}$ BSE

$$\begin{aligned}\Gamma(p; P) &= -\frac{4}{3} \int_q^\Lambda \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu \chi(q; P) \gamma_\nu, \\ \chi(q; P) &= S(q_+) \Gamma(q; P) S(q_-),\end{aligned}\quad (1)$$

on the one hand, where the semicolon separates four-vector arguments, and the rainbow-truncated quark DSE

$$\begin{aligned}S(p)^{-1} &= (i\gamma \cdot p + m_q) + \Sigma(p), \\ \Sigma(p) &= \frac{4}{3} \int_q^\Lambda \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu S(q) \gamma_\nu\end{aligned}\quad (2)$$

on the other hand.

The solution of the BSE is the Bethe-Salpeter amplitude (BSA) $\Gamma(q; P)$ which, combined with two dressed

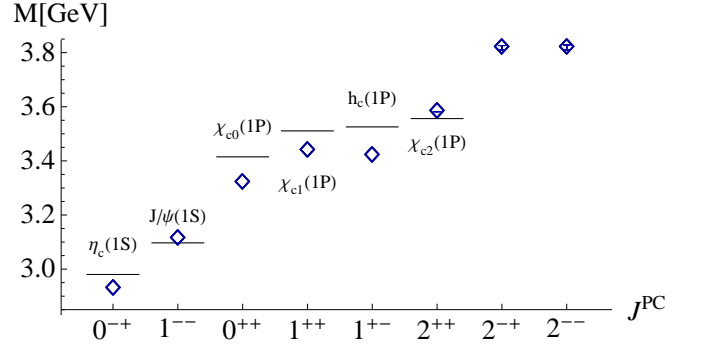


FIG. 2. (Color online) The charmonium ground-state spectrum compared to experimental data.

quark propagators $S(q_+)$ and $S(q_-)$ gives the “Bethe-Salpeter wave function” $\chi(q; P)$. Note that in the case of spin $J > 0$ the BSA carries J open Lorentz indices (for details see [41]), which are omitted here for simplicity together with the BSA’s Dirac and flavor indices. The factor $\frac{4}{3}$ comes from the color trace, $D_{\mu\nu}^f(p-q)$ is the free gluon propagator, γ_ν is the bare quark-gluon vertex, $\mathcal{G}((p-q)^2)$ is the effective interaction specified in detail below, and the (anti)quark momenta are $q_+ = q + \eta P$ and $q_- = q - (1-\eta)P$. $\eta \in [0, 1]$ is referred to as the momentum partitioning parameter and is usually set to 1/2 for systems of equal-mass constituents, which we do as well. $\int_q^\Lambda = \int d^4q / (2\pi)^4$ represents a translationally invariant regularization of the integral, with the regularization scale Λ [29]. $\Sigma(p)$ denotes the quark self energy and m_q the current-quark mass. The solution for the quark propagator $S(p)$ requires a renormalization procedure, the details of which can be found together with the general structure of both the BSE and quark DSE in [29, 31].

For the homogeneous, ladder-truncated $q\bar{q}$ BSE numerical solution methods have been improved in recent years [46] and also cross-checked with the closely related approach to hadron phenomenology via the corresponding inhomogeneous vertex BSEs, see e.g., [47–49]. Regarding the numerical solution of the quark DSE (2) we note that in order to be able to subsequently and consistently solve the BSE numerically, the propagator must be known for quark four-momenta whose squares lie inside a parabola-shaped region of the complex p^2 plane (for a more detailed discussion, see e.g., the appendix of Ref. [41]). As a consequence, in particular for heavy quarks a reliable numerical approach to the quark DSE is needed and we refer the reader to [50] for the details of our particular solution method. Furthermore, it is important to note here that the analytical structure of the quark propagator can place restrictions in terms of an upper bound on the bound-state masses of mesons accessible via standard numerical methods (see [51] for a detailed discussion and an initial step towards a full numerical treatment of such a situation). A route different to the one described in [51] is to extrapolate data

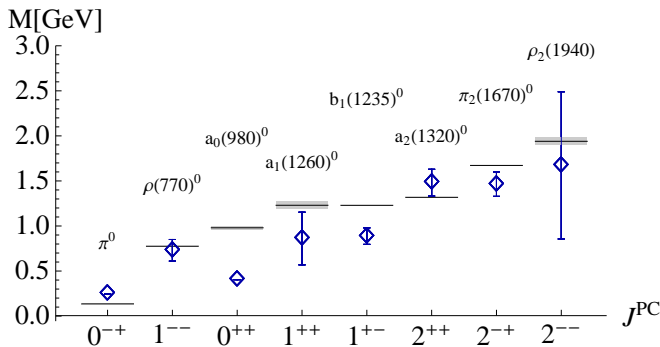


FIG. 3. (Color online) The light-isovector ground-state spectrum compared to experimental data.

obtained from the homogeneous BSE in the accessible region to the inaccessible point of interest. This is the approach we use here when necessary, and we give the details of our procedure in the appendix.

Once RL truncation has been chosen, the integral-equation kernels of (1) and (2) are essentially characterized by an effective interaction $\mathcal{G}(s)$, $s := (p - q)^2$. The parameterization of Ref. [31] reads

$$\frac{\mathcal{G}(s)}{s} = \frac{4\pi^2 D}{\omega^6} s e^{-s/\omega^2} + \frac{4\pi \gamma_m \pi \mathcal{F}(s)}{1/2 \ln[\tau + (1 + s/\Lambda_{\text{QCD}}^2)]}. \quad (3)$$

This form produces the correct perturbative limit, i.e. it preserves the one-loop renormalization group behavior of QCD for solutions of the quark DSE. As given in [31], $\mathcal{F}(s) = [1 - \exp(-s/[4m_t^2])]/s$, $m_t = 0.5$ GeV, $\tau = e^2 - 1$, $N_f = 4$, $\Lambda_{\text{QCD}}^{N_f=4} = 0.234$ GeV, and $\gamma_m = 12/(33 - 2N_f)$. The motivation for this function, which mimics the behavior of the product of quark-gluon vertex and gluon propagator, is mainly phenomenological. While currently debated on principle grounds (e.g. [52, 53]) the impact of its particular form in the far IR on meson masses is expected to be small (see also [54] for an exploratory study in this direction).

In [31], together with the current-quark mass m_q , the parameters ω and D were fitted to pion observables and the chiral condensate. In this way, this effective coupling provided the correct amount of dynamical chiral symmetry breaking as well as quark confinement via the absence of a Lehmann representation for the dressed quark propagator. Also, from the results of the fitting procedure in [31] it was apparent that for a fixed current-quark mass one can obtain a good description of light pseudoscalar and vector meson masses and decay constants by keeping the product $D \times \omega = 0.372$ GeV³ fixed and varying ω in the range $[0.3, 0.5]$ GeV. In particular, these observables were independent of ω , which thus defines a one-parameter model. Later, it was found that such a negligible dependence on ω is the characteristic of a ground state, while excitations—both radial and orbital—in general depend strongly on ω [30, 32]. As a result, it is possible to fix also ω to phenomenology, which we do in the

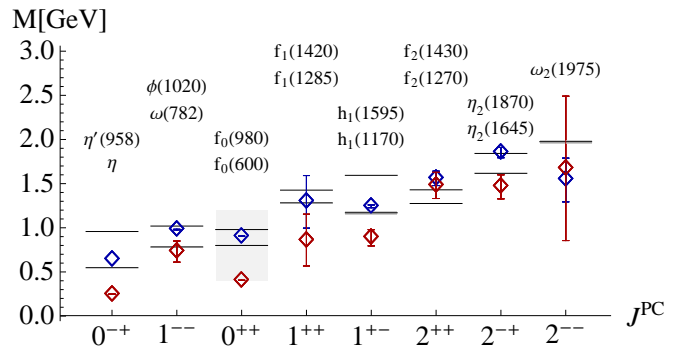


FIG. 4. (Color online) The light-isoscalar ground-state spectrum compared to experimental data. The red symbols correspond to the $n\bar{n}$ -states of our ideally mixed setup, the blue symbols denote pure $s\bar{s}$ states.

present study, albeit without fine-tuning the parameters any further, which is both besides the point as well as beyond the scope of this work.

IV. BOTTOMONIUM

With $D \times \omega$ fixed to the original value of 0.372 GeV³ and taking into account the trend visible in earlier computations of parts of the bottomonium spectrum e.g., in [41, 49], we varied $\omega > 0.5$ GeV and found good agreement with all experimentally known bottomonium ground states at $\omega = 0.61$ GeV via a least-squares fit of the masses of the states with $J^P = 0^-, 0^+, 1^-, 1^+$, and 2^+ . The masses for the states with $J^P = 2^-$ are thus predictions of the model. The results of our calculations are summarized in the first row of Tab. I and compared to the available experimental data in Fig. 1. Our numerical uncertainties are smaller than the sizes of the symbols in the figure for all cases.

These results for the masses show that already without fine-tuning we have achieved agreement on the level of 3 per-mill for all states considered, which is rather remarkable. In addition to the excellent overall agreement we also achieve reasonable agreement with e.g., the experimental value of 69 MeV for the hyperfine splitting between the 0^{-+} and 1^{-+} ground states, which is reproduced to 20%.

Furthermore we computed the leptonic decay constants of the pseudoscalar and vector bottomonium states and collect our results together with a comparison to experimental numbers (where available) in Tab. II. While the experimental value for f_{η_b} is yet unknown, we arrive within 4% of the experimental value for f_{Υ} . This latter observation is remarkable in particular, since the computation of the leptonic decay constants goes beyond spectroscopy in that it involves also the BSAs of the states under investigation. This means that the structure of the meson as described by the BSA is captured reasonably by our setup as well.

TABLE I. Calculated meson masses in MeV (rounded) for the four quark-mass values with extrapolation uncertainties given in brackets where applicable. A comparison to experimental data is given in Figs. 1 - 4. Note that our results for $\bar{n}n$ correspond to both light isovector and isoscalar states.

J^{PC}	0^{-+}	1^{--}	0^{++}	1^{++}	1^{+-}	2^{++}	2^{-+}	2^{--}
$\bar{b}b$	9405	9488	9831	9878	9873	9927	10184(8)	10188(8)
$\bar{c}c$	2928	3111	3321	3437	3421	3582	3818(8)	3818(9)
$\bar{s}s$	637	980(3)	904	1293(299)	1244(15)	1560(81)	1852(58)	1541(247)
$\bar{n}n$	248	730(120)	405(1)	861(292)	886(92)	1480(149)	1465(136)	1673(817)

TABLE II. Calculated pseudoscalar and vector meson decay constants in MeV (rounded) compared to experimental data [55], where available.

J^{PC}	0^{-+}	1^{--}
$\bar{b}b$	708	687
Experiment	—	715
$\bar{c}c$	399	448
Experiment	361	416
$\bar{s}s$	173	237^{+4}_{-3}
Experiment	—	227
$\bar{n}n$ isovector	108	276^{+61}_{-113}
Experiment	131	221

V. CHARMONIUM AND LIGHT MESONS

While the main point of interest in the present work is the compatibility of our ansatz with the ground state masses and decay constants of the bottomonium system, it is natural to ask how the same parameter set performs for charmonium as well as strange and light quark masses. We thus present the corresponding results in the remaining rows of Tabs. I and II and compare to available experimental data in Fig. 2 for charmonium and Figs. 3 and 4 for states made out of light and strange (anti)quarks.

It is important to note here that in our present truncation there is no flavor mixing, i.e., all states are thus *ideally mixed* and consequently *a priori* can be expected to correspond to experimental states only in the appropriate cases. Note also that we work in the isospin-symmetric limit. In our tables we therefore list our results for pure $\bar{s}s$ and $\bar{n}n$ states, where in the usual notation n labels light quark flavors. While one could apply simple flavor-mixing rules to our results [56] we do not attempt this here to maintain the clarity of our results as well as the simplicity of both the model and the intention of our work. It is not aimed at a perfect description of light meson masses; in fact, such an outcome cannot be expected of our present study, since RL truncation oversimplifies the structure of the quark-gluon vertex for light and strange quarks (and apparently to some degree also for charm quarks). However, it appears that a set of

results such as the present one can in the future be reconciled with experiment upon proper inclusion of corrections beyond RL truncation as they have been explored in the recent literature, e.g., [57, 58].

Regarding our charmonium results we observe an overall pattern of agreement with experimental data although some deficiencies are visible compared to the bottomonium case. Most notably the scalar and axial-vector masses are underestimated while the hyperfine splitting is overestimated. Our results for the pseudoscalar and vector leptonic decay constants are off 11 and 8% of the experimental numbers, which is about twice as much as for bottomonium.

For the strange and light quark cases we present our results compared to isoscalar and isovector states as listed by the PDG [58]. Note that our results for $\bar{n}n$ as listed in Tab. I correspond to both light isovector and isoscalar states. Due to the setup of the present study, our observations here can reasonably only be of a general nature. For the isovector case one observes a well-known pattern from the literature, namely that pseudoscalar and vector meson masses compare well to experiment, while scalar and axial-vector states are substantially underestimated [32]; for tensor mesons the situation is better [41]. Also for the decay constants, our model provides a reasonable description of the data, even in its present form.

For the isoscalar case, the situation is naturally more complex. Still, the overall impression is the same as described for the isovector case above except for the pseudoscalar mesons, whose flavor composition cannot be described in RL truncation.

VI. CONCLUSIONS AND OUTLOOK

We have reported a study of the bottomonium ground-state meson masses using and adjusting a well-established rainbow-ladder truncated effective-interaction setup of the Bethe-Salpeter-equation. Our goal was to provide—without fine-tuning—an initial test of the potential such an approach holds to describe experimental data in a reasonable fashion. Our results show that such a description is indeed possible and beyond a surprisingly good match in bottomonium also provides a reasonable description of charmonium and a consistent picture for meson masses containing light and strange

(anti)quarks.

This is an encouraging first step towards a comprehensive study of mesons in this approach. Further steps will involve fine-tuning but will also have to include appropriate corrections beyond RL truncation to provide the mechanism for a reconciliation of the deficiencies apparent in our present results for lower quark masses. Together with an anchor of the model in the heavy-quark domain such as the one exemplified here this will lead to a successful model description of hadrons.

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Appendix: Technicalities

As mentioned in Sec. III and described in detail in the appendix of Ref. [41] the Euclidean-space treatment of the meson BSE demands considerable conceptual and numerical care. In particular, for the reasons given in Sec. III one in some cases of higher-lying meson masses in our present approach has to resort to extrapolation techniques. In [41] a straight-forward method was used, which we have since improved upon; our refined method is reported in the following.

In the standard approach, the homogeneous BSE is solved as an eigenvalue equation, where the on-shell point (and thus the mass of the meson in question) is reached if the eigenvalue $\lambda(P^2 = M^2) = 1$ (for a detailed discussion of BSE eigenvalues, see [46]). As discussed in detail

in [49], this eigenvalue and its dependence on the total-momentum squared is deeply connected to the bound-state poles that appear in the corresponding four-point function,

$$\frac{\lambda(P^2)}{1 - \lambda(P^2)} = \frac{r}{P^2 + M^2} + \text{corrections} , \quad (\text{A.1})$$

where r denotes the residue at the pole corresponding to a particle of mass M .

If the corrections in Eq. (A.1) are neglected, the combination $\frac{1 - \lambda(P^2)}{\lambda(P^2)}$ becomes linear in P^2 , which was used for the extrapolations done in [49]. However, more reliable results can be obtained if the corrections are taken into account. In this work, we assume corrections of polynomial form,

$$\text{corrections} = \sum_{i=1}^N (P^2)^i c_i , \quad (\text{A.2})$$

with constants c_i . These constants c_i , the residue r as well as the resulting masses M are obtained in a straightforward fit to calculated values of the function $\frac{\lambda(P^2)}{1 - \lambda(P^2)}$ in the range of P^2 that can be accessed directly. In order to estimate the uncertainty of the extrapolation, the polynomial order N of the corrections is varied from $N = 1$ to 6, and our final result is computed from the arithmetic mean of this sample; the error bars are given by the spread of the largest and smallest value.

A similar method is used for the extrapolation of the decay constants f of the light and strange vector states. In these cases, polynomials of degree 3, 4, and 5 have been fitted to the available values of $f(\sqrt{-P^2}) \times \sqrt{-P^2}$ and extrapolated to $\sqrt{-P^2} = M$. Again, the average of the three resulting values is quoted as final result, and the uncertainties are estimated from the differences between the average and the largest and smallest value, respectively.

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- [1] N. Brambilla *et al.* (Quarkonium Working Group), hep-ph/0412158.
 - [2] N. Brambilla *et al.*, Eur. Phys. J. **C71**, 1534 (2010).
 - [3] C. S. Fischer, J. Phys. G **32**, R253 (2006).
 - [4] C. D. Roberts, M. S. Bhagwat, A. Holl, and S. V. Wright, Eur. Phys. J. Special Topics **140**, 53 (2007).
 - [5] N. Hu *et al.*, Acta Sci. Naturalium Universitatis Pekinensis **12**, 213 (1966).
 - [6] N. Hu *et al.*, Acta Sci. Naturalium Universitatis Pekinensis **12**, 209 (1966).
 - [7] C. H. Llewellyn-Smith, Ann. Phys. **53**, 521 (1969).
 - [8] G. Eichmann, A. Krassnigg, M. Schwinzerl, and R. Alkofer, Annals Phys. **323**, 2505 (2008).
 - [9] D. Nicmorus, G. Eichmann, A. Krassnigg, and R. Alkofer, Phys. Rev. D **80**, 054028 (2009).
 - [10] G. Eichmann, R. Alkofer, C. S. Fischer, A. Krassnigg, and D. Nicmorus, 1010.0206 [hep-ph].
 - [11] G. Eichmann, R. Alkofer, A. Krassnigg, and D. Nicmorus, Phys. Rev. Lett. **104**, 201601 (2010).
 - [12] G. Eichmann, *Hadron properties from QCD bound-state equations*, Ph.D. thesis, University of Graz (2009), 0909.0703 [hep-ph].
 - [13] M. S. Bhagwat, A. Holl, A. Krassnigg, C. D. Roberts, and P. C. Tandy, Phys. Rev. C **70**, 035205 (2004).
 - [14] G. Eichmann, R. Alkofer, I. C. Cloet, A. Krassnigg, and C. D. Roberts, Phys. Rev. C **77**, 042202(R) (2008).
 - [15] G. Eichmann, I. C. Cloet, R. Alkofer, A. Krassnigg, and C. D. Roberts, Phys. Rev. C **79**, 012202(R) (2009).
 - [16] C. Popovici, P. Watson, and H. Reinhardt, Phys. Rev. D **81**, 105011 (2010).
 - [17] C. Popovici, P. Watson, and H. Reinhardt, Phys. Rev. **D83**, 125018 (2011).
 - [18] C. Popovici, *Quark sector of Coulomb gauge Quantum Chromodynamics*, Ph.D. thesis, University of Tuebingen (2011), 1106.0691 [hep-ph].
 - [19] T. Maskawa and H. Nakajima,

- Prog. Theor. Phys. **52**, 1326 (1974).
- [20] K.-I. Aoki, M. Bando, T. Kugo, M. G. Mitchard, and H. Nakatani, Prog. Theor. Phys. **84**, 683 (1990).
 - [21] T. Kugo and M. G. Mitchard, Phys. Lett. B **282**, 162 (1992).
 - [22] M. Bando, M. Harada, and T. Kugo, Prog. Theor. Phys. **91**, 927 (1994).
 - [23] H. J. Munczek, Phys. Rev. D **52**, 4736 (1995).
 - [24] P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B **420**, 267 (1998).
 - [25] P. Maris and P. C. Tandy, Phys. Rev. C **61**, 045202 (2000).
 - [26] P. Maris and P. C. Tandy, Phys. Rev. C **62**, 055204 (2000).
 - [27] R. Williams and C. S. Fischer, 0912.3711 [hep-ph].
 - [28] L. Chang and C. D. Roberts, 1003.5006 [nucl-th].
 - [29] P. Maris and C. D. Roberts, Phys. Rev. C **56**, 3369 (1997).
 - [30] A. Holl, A. Krassnigg, and C. D. Roberts, Phys. Rev. C **70**, 042203(R) (2004).
 - [31] P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
 - [32] A. Krassnigg, Phys. Rev. D **80**, 114010 (2009).
 - [33] P. Maris and P. C. Tandy, Nucl. Phys. Proc. Suppl. **161**, 136 (2006).
 - [34] A. Holl, A. Krassnigg, P. Maris, C. D. Roberts, and S. V. Wright, Phys. Rev. C **71**, 065204 (2005).
 - [35] M. S. Bhagwat and P. Maris, Phys. Rev. C **77**, 025203 (2008).
 - [36] G. Eichmann, Phys. Rev. D **84**, 014014 (2011).
 - [37] V. Mader, G. Eichmann, M. Blank, and A. Krassnigg, Phys. Rev. D **84**, 034012 (2011).
 - [38] T. T. Nguyen, *Aspects of non-perturbative QCD for meson physics*, Ph.D. thesis, Kent State University (2010).
 - [39] P. C. Tandy, 1011.5250 [nucl-th].
 - [40] T. Nguyen, A. Bashir, C. D. Roberts, and P. C. Tandy, Phys. Rev. C **83**, 062201 (2011).
 - [41] A. Krassnigg and M. Blank, Phys. Rev. D **83**, 096006 (2011).
 - [42] M. Blank and A. Krassnigg, Phys. Rev. D **82**, 034006 (2010).
 - [43] P. Maris, AIP Conf. Proc. **892**, 65 (2007).
 - [44] N. Souchlas, *Quark dynamics and constituent masses in heavy quark systems*, Ph.D. thesis, Kent State University (2009).
 - [45] N. Souchlas and D. Stratakis, Phys. Rev. D **81**, 114019 (2010).
 - [46] M. Blank and A. Krassnigg, Comput. Phys. Commun. **182**, 1391 (2011).
 - [47] M. S. Bhagwat, A. Hoell, A. Krassnigg, C. D. Roberts, and S. V. Wright, Few-Body Syst. **40**, 209 (2007).
 - [48] M. Blank and A. Krassnigg, AIP Conf. Proc. **1343**, 349 (2011), to appear in the proceedings of “Quark Confinement and the Hadron Spectrum IX”.
 - [49] M. Blank, *Properties of quarks and mesons in the Dyson-Schwinger/Bethe-Salpeter approach*, Ph.D. thesis, University of Graz (2011), 1106.4843 [hep-ph].
 - [50] A. Krassnigg, PoS **Confinement8**, 75 (2009).
 - [51] M. S. Bhagwat, M. A. Pichowsky, and P. C. Tandy, Phys. Rev. D **67**, 054019 (2003).
 - [52] C. S. Fischer, A. Maas, and J. M. Pawłowski, Annals Phys. **324**, 2408 (2008).
 - [53] D. Binosi and J. Papavassiliou, Phys. Rept. **479**, 1 (2009).
 - [54] M. Blank, A. Krassnigg, and A. Maas, Phys. Rev. D **83**, 034020 (2011).
 - [55] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010) and 2011 partial update for the 2012 edition.
 - [56] A. Holl, A. Krassnigg, C. D. Roberts, and S. V. Wright, Int. J. Mod. Phys. A **20**, 1778 (2005).
 - [57] C. S. Fischer and R. Williams, Phys. Rev. Lett. **103**, 122001 (2009).
 - [58] L. Chang and C. D. Roberts, Phys. Rev. Lett. **103**, 081601 (2009).